

Water balance modelling of a cropped soil: deterministic and stochastic approaches

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Abstract A capacity-based simulation model has been developed to predict components of the field water balance. The model requires daily information such as the atmospheric evaporative demand and rain events, maximum-available soil water storage (MAWS) and root growth characteristics as well as crop and soil factors. Actual evapotranspiration is modelled using Eagleman's parameterization. The model was tested for upland rice grown on a 3500 m² field plot and on an undisturbed lysimeter, both located at Bouaké, Ivory Coast. Good agreement between calculations and observations of actual evapotranspiration (AET) and drainage (*D*) measured at 15 sites equipped with neutron access tubes and tensiometers demonstrates the reliability of the model. This is also independently confirmed against the lysimeter data. The model was used to study the influence of the soil spatial variability on AET and *D* at the bottom of the soil profile. This was achieved by considering MAWS as a stochastic process defined by its probability density and covariance functions. Therefore, the model response for the entire field is expressed in terms of probability. It is shown that a deterministic simulation obtained with the mean value of MAWS may lead to unacceptable predictions of the field average AET and *D* values, the difference with the stochastic mean strongly depending on the degree of variability and on the water supply conditions.

INTRODUCTION

Water flow into the unsaturated zone is of great importance in agricultural and hydrological applications, especially in dry tropical zones where the level of water consumption by crops is an important element in the determination of yield. In general, the traditional approach for modelling water processes in soil systems is to consider the field as a homogeneous medium and to apply theories and results valid for laboratory columns. During the last two

decades a large number of such mechanistic-deterministic models have been developed to simulate the transient unsaturated water flow combined with uptake by plant roots; see for instance reviews by De Jong (1981) and Feddes *et al.* (1988). Such detailed models are excellent research tools. However, their large data requirements strongly limit their use as management tools. However, less-detailed water budget models, called functional models by Addiscott & Wagenet (1985), that are physically reasonable and computationally efficient, remain useful. This is especially so where the available field data are limited and/or difficult to obtain.

In addition, it has been known for many years that local field properties can exhibit very pronounced spatial fluctuations. In such heterogeneous media the transfer phenomena may be described as erratic processes susceptible to quantitative characterization by stochastic models.

The purpose of this paper is twofold. Firstly, to assess a functional deterministic water budget model by comparison with experimental field data. Secondly, to develop a stochastic version of this model while attempting to answer the following questions: (a) is a stochastic approach necessary to simulate water budget at the field-scale? (b) if it is not, under which constraints if any, may deterministic modelling be sufficient?

THE DETERMINISTIC MODEL

Presentation of the model

We start with a deterministic-capacity based model initially developed by Franquin & Forest (1977) and then modified by Chopart & Siband (1988) and by Chopart & Vaudin (1990) to take into account the temporal evolution of the rooting depth. In this model the soil profile is viewed as having two reservoirs:

- (a) The first one is a time dependent reservoir attached to the root zone. It is limited on day J by the root front $ZR(J)$ (cm) and by the maximum quantity of water available for the crop per unit depth of soil, MAWS (mm of water/cm of soil). Its maximum storage capacity on day J is then $SR_{max}(J) = ZR(J) \times MAWS$.
- (b) The second is a fixed reservoir limited by the maximum depth of the soil profile ZD (cm) and by MAWS; its maximum capacity being $S_{max} = ZD \times MAWS$.

Integrating explicitly with time the mass conservation equation between the soil surface ($Z = 0$) and the depths $ZR(J)$ and ZD leads to:

$$\begin{aligned}
 SR(J+1) = & SR(J) + R(J) + \{ZR(J+1) - ZR(J)\} \times MAWS \\
 & - DR(J) - AET
 \end{aligned}
 \tag{1}$$

and

$$S(J+1) = S(J) + R(J) - D(J) - AET(J)
 \tag{2}$$

where $SR(J)$ and $S(J)$ are the actual water storage in the root zone, and the entire soil profile respectively. They must be such that:

$$\begin{aligned} 0 < SR(J) < SR_{\max}(J) \\ \text{and} \\ 0 < S(J) < S_{\max} \end{aligned} \quad (3)$$

During day J , $R(J)$ is the amount of rainfall and/or irrigation; $DR(J)$ and $D(J)$ are the water losses below $ZR(J)$ and ZD respectively. Their estimates are based upon the filling of the two reservoirs in relation to their maximum capacity values:

$$\begin{aligned} DR(J) &= \max \{0, SR(J) + R(J) - SR_{\max}(J)\} \\ \text{and} \\ D(J) &= \max \{D, S(J) + R(J) - S_{\max}\} \end{aligned} \quad (4)$$

In equations (1) and (2), $AET(J)$ is the actual evapotranspiration which drives the model. It is modelled through Eagleman's (1971) empirical approach modified by Forest (Forest & Reyniers, 1986) to relate $AET(J)$ to the maximum evapotranspiration $MET(J)$, following the expression:

$$AET(J) = a + b \cdot SMR(J) + c[SMR(J)]^2 + d[SMR(J)]^3 \quad (5)$$

with:

$$\begin{aligned} a &= 0.732 - 0.05 \cdot MET(J); & b &= 4.97 \cdot MET(J) - 0.661 [MET(J)]^2 \\ c &= 8.57 \cdot MET(J) + 1.56 [MET(J)]^2; & d &= 4.35 \cdot MET(J) - 0.880[MET(J)]^2 \end{aligned}$$

Here $SMR(J)$ is the soil moisture ratio, simply defined as:

$$SMR(J) = \{SR(J) + R(J) - DR(J)\} / SR_{\max} \quad (6)$$

In equation (5), the maximum evapotranspiration is expressed here as:

$$MET(J) = \max \{KC(J), KS(J)\} \times PET(J) \quad (7)$$

where $PET(J)$ is the climatic evaporative demand of the day J , $KC(J)$ and $KS(J)$ are the crop and soil factors accounting for the transpiration and evaporation, respectively.

The relation between AET and MET for different values of SMR is plotted in Fig. 1.

In addition to the requirement of the initial conditions ($SR(0)$, $S(0)$, $ZR(0)$) the model requires *a priori* knowledge of the following: ZD , $MAWS$, $KS(J)$ for the soil, $ZR(J)$, $KC(J)$ for the crop and the weather information $R(J)$ and $PET(J)$. It should be noted that the rooting depth cannot be greater than the wetting front depth $ZWF(J)$ as calculated by:

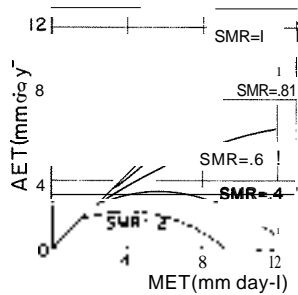


Fig. 1 The calculated evapotranspiration, AET, as a function of maximum evapotranspiration, MET, for different soil moisture ratios, SMR.

$$ZWF(J) = \{S(J) + R(J)\} / MAWS \quad (8)$$

Thus, penetration by roots must wait until the soil water has increased at that depth to allow the root growth.

Given the weather data for day J , since $SR(J)$ and $S(J)$ are known at the beginning of that day, equation (4) gives the water loss by drainage below the two reservoirs. Equations (5) and (6) then give the actual evapotranspiration as well as the actual water storage $SR(J+1)$ and $S(J+1)$. These are calculated according to equations (1) and (2) with the constraint (3). The corresponding user-friendly but uncompiled version of the model written in BASIC and called PROBE for PROgramme de Bilans en Eau (Chopart & Siband, 1988) is available upon request.

Validation of the model

In order to validate the model, the results of two experiments are considered: The first a field experiment and the other on lysimeter.

Field experiment This was conducted in 1986 at the IDESSA (Institut des Savanes) agricultural station in Bouaké, Ivory Coast ($7^{\circ}40'N$ latitude and $5^{\circ}05'W$ longitude). The general purpose of this experiment was to study the influence of sowing density on water use and fertilizer consumption by upland rice (*Oriza sativa*, cv IRA 144, Japonica type). Only the experimental results dealing with water use are considered in this paper to allow comparison between the experimental results and the model predictions.

The climate of the area is of Sudano-Guinean type, with an average annual rainfall of 1150 mm and two rainy seasons of April-June and August-October which are separated by a short dry season. The soil is classified as an Alfisol.

The experimental set-up consisted of a 13.5 by 34 m field divided into 16 blocks. Within each block, three treatments were located randomly. These corresponded to sowing densities of 89 plants m^{-2} (T1), 22 plants m^{-2}

(TI) and 8 plants m^{-2} (TI). In order to enlarge the spectrum of rainfall conditions, rice was sown on two dates: 5 August 1986 for blocks 1 to VIII and 12 August 1986 for blocks IX to XVI. The harvest dates were 24 November and 2 December respectively. All the blocks received about the same amount of water, except blocks VII, VIII, IX and X which were irrigated during the last two weeks before harvest. During this period, the other blocks experienced drought stress.

Five blocks, each with three treatments, were equipped with a neutron access tube down to 1.1 m. Also each had four tensiometers vertically installed at 0.6, 0.8, 1.0 and 1.2 m deep. These were all connected to mercury manometers. A raingauge was located close to each neutron access tube for measurement of rainfall and irrigation on a daily basis. At each measurement site an internal drainage test (Hillel *et al.*, 1972) had been performed during the previous two dry seasons in order to determine the unsaturated hydraulic conductivity at $ZD = 0.9$ m.

All the observations were analyzed site by site in order to infer the experimentally measured water balance components AET_m and D_m from the equation:

$$\Delta S_m = R - AET_m - D_m \quad (9)$$

where ΔS_m is the variation of soil water storage between two consecutive neutron measurements and D_m is the drainage at $ZD = 0.9$ m, estimated from Darcy's law. The hydraulic gradient was obtained from the tensiometers at depths 0.8 and 1.0 m.

Lysimeter study The same rice cultivar was grown as in treatment TI on an undisturbed drainage lysimeter of 4 m² surface area and 0.7 m deep. Non-limiting soil water conditions were maintained by applying irrigation in order to maintain drainage at the base of the lysimeter. Measurements of the inputs (R) and loss (D_m) gave the evapotranspiration (equation (9) with $\Delta S_m = 0$). To minimize the possibility of border effects, the lysimeter was surrounded by a buffer zone that was cultivated and managed in the same way.

Model inputs The different parameters and variables necessary to run the model for these specific conditions are briefly listed below.

- (a) *Plant parameters* Observations of the growth of the root system led to the following relation between ZR and the time after sowing:

$$\begin{aligned} ZR(J) &= 12.5 + 0.9 J & \text{for } J < 75 \text{ days} \\ ZR(J) &= 80 \text{ cm} & \text{for } J > 75 \text{ days} \end{aligned} \quad (10)$$

Rice factors $KC(J)$ for the three treatments were inferred from an experiment performed in the previous year on the same field under well-watered conditions (Chopart & Vauclin, 1990).

- (b) *Soil parameters* At each site the maximum available soil water

storage, MAWS, was determined as the difference between the maximum and the minimum (at harvest time of the rice) soil water storages as measured by the neutron probe during the growing season. The corresponding values were found to be normally distributed with a mean value of 0.58 mm cm^{-1} and a standard deviation of 0.055 mm cm^{-1} . Due to a lack of experimental data about soil evaporation, an empirical function between KS and days after the soil was last wetted (DAR) was derived by regression between the measured and calculated values of AET, at one site during the first 30 days of growth when KC was smaller than KS . This gave

$$KS = \exp(-0.76 \text{ DAR}) \quad (11)$$

where DAR stands for days after rain. $DAR = 0$ is the day of the rain.

- (c) *Weather variables* Daily rainfall and irrigation were measured at each site and potential evaporation rates (PET) were calculated using the modified Penman equation proposed by Monteny *et al.* (1981).

Comparison between **experimental** and calculated results The computer model was run for each site with its requisite soil, plant and weather data. For the lysimeter, the calculations were made with $ZD = 0.7 \text{ m}$ and $MAWS = 0.77 \text{ mm cm}^{-1}$. The model began on the day after sowing and the time step was one day. The computation time on a personal computer with clock speed 8 MHz and a 80286 processor was about 0.375 s day^{-1} . It should be emphasized that no attempt was made to obtain a best fit between computations and experimental results.

Predicted and measured components of the seasonal water balance are presented in Table 1 for all the sites and for the lysimeter as well. Figure 2 presents these results for one site. The calculated and measured patterns of AET averaged over five 1-day periods, and the cumulative D values over five 1-day periods are given, as well as the estimates of the potential evaporation (PET) and maximum evapotranspiration (MET) rates. These results show that the predictions of the actual evapotranspiration and drainage matched very well the lysimeter data (Table 1) for which experimental errors are quite small. Although less-satisfying, the agreement between the computed and measured water balance components for the field remained acceptable. The root-mean-square error between the two was about 16% for AET, and 19% for D which appears very fair, in view of the uncertainties associated with the field measurements, especially in the estimate of the flux at the bottom of the soil profile by Darcy's law.

It can also be seen in Fig. 2 that the model correctly simulated both the short and long term variations in the water balance components. These were induced by both the weather, and phenological development of rice. The same behaviour was observed at all other sites and there was no significant long-term drift in the predictions.

Analysis of the results obtained for all field sites gave the following linear regressions between the calculated and measured values:

Table 1 Deterministic approach: comparison between the measured (subscript m) and calculated (subscript c) components of the seasonal water balance for the field (sites 1 to 15) and the lysimeter. All the values are in mm' of water. () indicates the irrigated sites*

SITES	R (mm)	AET _m	AET _c	D _m	De
1	397	275	293	161	132
2	457	280	318	192	161
3*	570	343	365	232	204
4	451	238	290	240	170
5	454	230	283	251	197
6	416	263	293	178	145
7	453	278	315	211	161
8*	566	308	362	256	210
9	440	242	271	224	183
10	474	243	286	240	198
11	474	238	283	254	211
12	438	240	285	235	180
13*	541	263	326	278	221
14	427	227	256	222	185
15	423	217	252	238	193
Lysimeter	427	292	284	135	132

$$AET_e (\text{mm day}^{-1}) = 0.965 AET_m + 0.284 \quad r = 0.856 \quad (n = 296)$$

$$De (\text{mm day}^{-1}) = 0.950 D_m - 0.153 \quad r = 0.885 \quad (n = 296)$$

Tests performed on the slopes and the intercepts of the regression lines showed that they are not significantly different from the 1:1 line at the 0.05 significance level. Because of the satisfactory agreement between observed and calculated values this model is a good candidate for studying the impact of spatial variability in soil parameters.

THE STOCHASTIC MODEL

Presentation of the approach

Here, the model previously presented is coupled with a statistical description of MAWS assimilated to a two-dimensional, second-order stationary, isotropic-stochastic process defined by its probability density and covariance functions. The resulting stochastic equations have been solved by the Monte-Carlo simulation technique.

The field has been discretized in 400 equally sized blocks. At each block, a value of MAWS has been generated by the Thinning Band Method of Mantoglou & Wilson (1982) modified by Munoz-Pardo *et al.* (1988). This leads to one run of the studied stochastic process for which the model is repeatedly run for each block. The statistical analysis of the outputs (AET

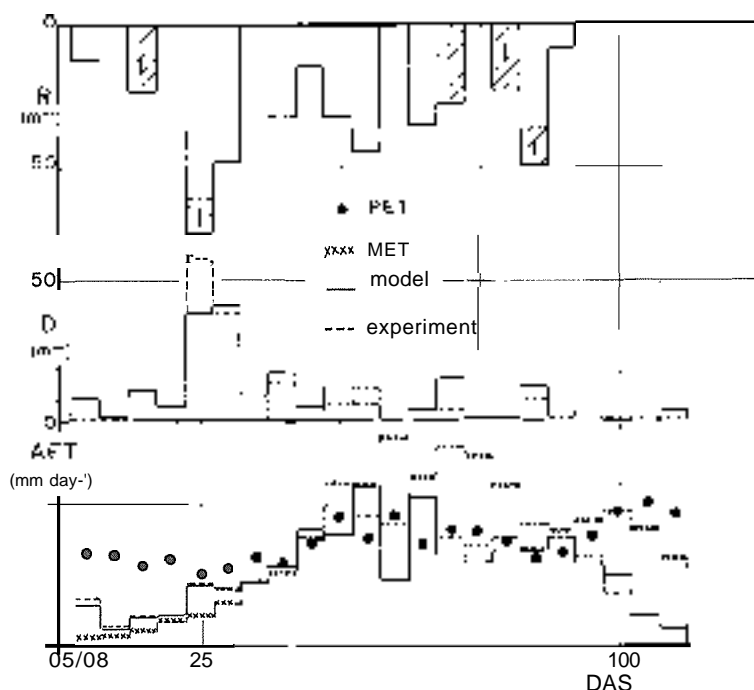


Fig. 2 Deterministic approach: comparison between the measured and calculated components of the water balance at one site. D is the drainage at 90 cm, AET is the actual evapotranspiration. The values of the maximum evapotranspiration (MET) and potential evaporation (PET) are also given along with the rainfall pattern and irrigation events (hatched areas).

and D) over the whole number of blocks defines the spatial statistics (e.g. mean value, variance) of the run. Reproducing the preceding steps for a large number of runs ($NR = 200$) allows the complete statistics of the variables of interest to be defined. More details on the approach can be found in Marchand (1988).

Results of stochastic simulations

As an example, Fig. 3 presents the statistics of one run of the cumulative AET and D calculated by considering the maximum available soil water storage as a random function: normally distributed (mean value = 0.58 mm cm^{-1} ; standard deviation = 0.22 mm cm^{-1}), with an isotropic exponential variogram (equivalent range = 9.45 m). While the distribution of the model input MAWS is symmetrical (Fig. 3(a)), the highly skewed distributions of AET and D (Fig. 3(c) and 3(d)) should be noted as well as the conservation of the spatial structure between the input and the outputs. It is worthwhile

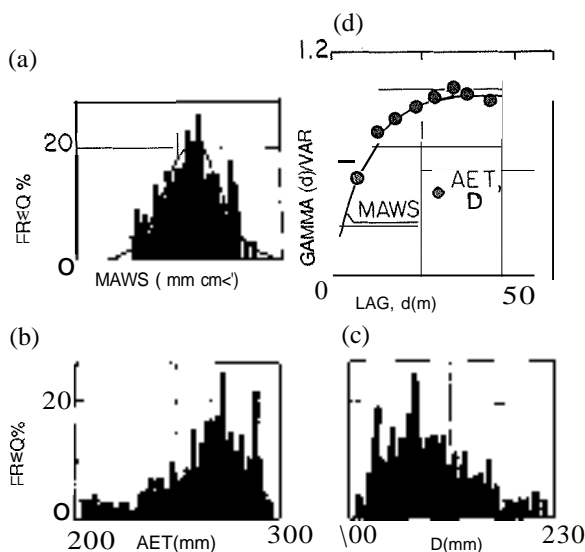


Fig. 3 Stochastic approach. Statistics of the model input (MAWS) and outputs (AET, D): probability density functions of (a) the maximum available soil water storage, MAWS, (b) the cumulative actual evapotranspiration AET and (c) drainage D. (d) Semi-variances as a function of distances between points, d for MAWS, AET and D.

to mention that this structure was indirectly confirmed two years later by analysing the spatial variations of cotton yield components measured at 108 locations within this field. The corresponding range (about 10 m) was found to be of the same order of magnitude as the range of MAWS used here (Vaudin & Chopart, 1990).

Figure 4 gives the values for each 10-day period of the stochastic mean (expected value), lower and upper quartiles for AET and D (mm day⁻¹). Depending on the rainfall, the evaporative demand and the moisture ratio in the root zone, it clearly appears that the values of AET and D are more or less influenced by the variability of MAWS. If a sensitive period in terms of water consumption takes place during a sensitive physiological period for the plant, the differences as explicitly given by the stochastic simulation may have a large influence on the overall behaviour of the culture.

The influence of the variability of MAWS on the AET and D values summed over the whole growing cycle is depicted in Fig. 5. The evolution of the stochastic means, the deterministic means calculated with the average value of MAWS, and the upper and lower quartiles are also represented as a function of the mean MAWS (Fig. 5(a) and 5(b)) and of the standard deviation of MAWS (Fig. 5(c) and 5(d)). It is shown that the difference between the stochastic and deterministic means increases with the soil variability, as well as the uncertainties in the estimates of AET and D . Note that 50% of these values lie outside the hatched areas in Fig. 5. This clearly

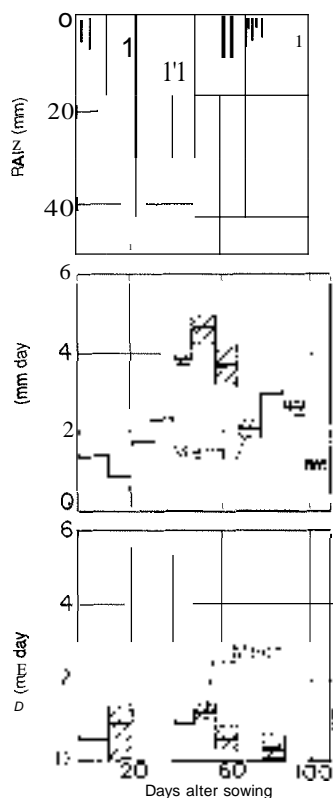


Fig. 4 Stochastic approach. Time evolution of calculated mean values of the actual evapotranspiration and drainage. 50% of the values lie inside the hatched areas.

illustrates the difficulty of selecting a few, but so-called representative sites of measurements of the water balance components, in heterogeneous fields.

It can also be noted that a deterministic simulation with a correct estimate of the mean value of MAWS (calculated from its probability density function) may lead to a fairly good estimation of the water budget components representative of the whole field. However, an erroneous estimate of this mean value in cases of either high variability or water stress, may lead to unacceptable predictions of the field average behaviour. In those situations, the stochastic approach is highly recommended.

CONCLUSIONS

To simulate water consumption by field crops, it is often necessary to use simplified representations of physical and physiological processes. This study demonstrates success with a capacity-based model which provided without calibration, acceptable local predictions of the water balance components. Its

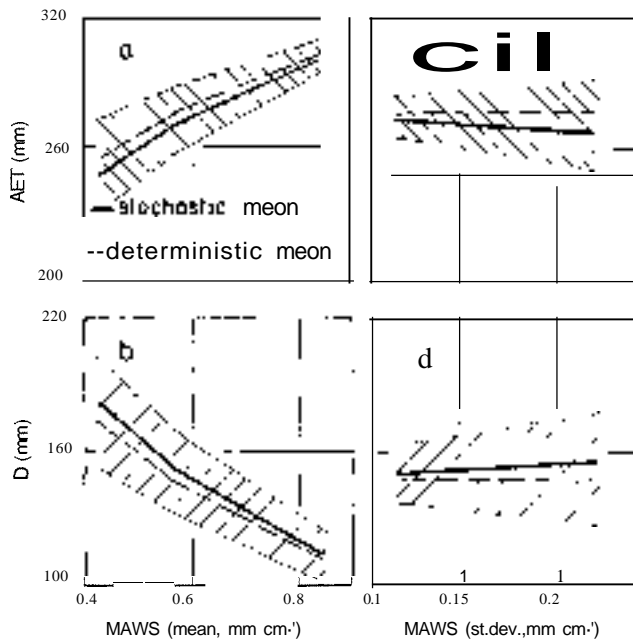


Fig. 5 Comparison between stochastic and deterministic means of the actual evapotranspiration and drainage for different values of the mean maximum available soil water storage (a and b) and the standard deviation (c and d). Fifty per cent of the values lie inside the hatched areas.

simplicity and computational efficiency make it valuable for studying the impact of spatial variability of soil properties. This was achieved by considering the maximum available soil water storage as a two-dimensional stochastic process, characterized by known probability density and covariance functions.

The resulting stochastic model gave very realistic responses of the entire field, expressed in probabilistic terms. Also the results suggest that the use of a pure deterministic approach, considering the field as an equivalent homogeneous medium may be questionable, depending on the level of variability and on the water supply conditions.

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